Charmless $\overline{B}_s \to VV$ decays in QCD factorization

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Abstract

The two body charmless decays of B_s meson to light vector mesons are analyzed within the framework of QCD factorization. This approach implies that the nonfactorizable corrections to different helicity amplitudes are not the same. The effective parameters a_i^h for helicity $h = 0, +, -$ states receive different nonfactorizable contributions and hence are helicity dependent, contrary to naive factorization approach where a_i^h are universal and polarization independent. The branching ratios for $\bar{B}_s \to VV$ decays are calculated and we find that branching ratios of some channels are of order 10^{-5}, which are measurable at future experiments. The transverse to total decay rate Γ_T/Γ is also evaluated and found to be very small for most decay modes, so, in charmless $\overline{B}_s \to VV$ decays, both light vector mesons tend to have zero helicity.

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1 Introduction

The charmless two-body B decays play a crucial role in determining the flavor parameters, especially the Cabibbo-Kobayashi-Maskawa(CKM) angels α, β and γ. With precise measurements of these parameters, we can explore CP violation which is described by the phase of the CKM matrix in the standard model(SM). Recently there have been remarkable progresses in the study of exclusive charmless B decays, both experimentally and theoretically. On the experimental aspect, many two-body non-leptonic charmless B decays have been observed by CLEO and B-factories at KEK and SLAC [1, 2] and more B decay channels will be measured with great precision in the near future. With the accumulation of data, SM can be tested in more detail. Theoretically, several novel methods have also been proposed to study the non-factorizable effects in the hadronic matrix elements, such as QCD factorization(QCDF) [3], the perturbative QCD method(pQCD) [4] and so on. Intensive investigations on hadronic charmless two-body $B_{u,d}$ decays with these methods have been studied in detail [5, 6, 7, 8, 9].

The extension of QCDF from $B_{u,d}$ decays to B_s decays has also been carried out by several authors [10, 11]. In principle, the physics of the B_s two-body hadronic decays is very similar to that for the B_d meson, except that the spectator d quark is replaced by the s quark. However, the problem is that B_s meson oscillates at a high frequency, and nonleptonic B_s decays have still remained elusive from observation. Unlike the $B_{u,d}$ mesons, the heavier B_s meson cannot be studied at the B-factories operating at the $\Upsilon(4s)$ resonance. But it is believed that in the forthcoming hadron colliders such as the Collider Detector at Fermilab (CDF), D0, DESY *ep* collider HERA-B, BTeV, and CERN Large Hadron Collider (LHCb), CP violation in B_s system can be observed with high accuracy. This makes the search for CP violation in the B_s system decays very interesting.

In the papers [11, 12], the authors have studied systematically the $B_s \to PP, PV$ decays(here P, V denote pseudoscalar and vector mesons respectively) with QCD factorization, and intensive phenomenological analysis has been made. Since the $B \to VV$ modes reveal dynamics of exclusive B meson decays more than the the $B \to PP$ and PV modes through the measurement of the magnitudes and the phases of various helicity amplitudes, in the present work we plan to make a detail study of $\overline{B}_s \to VV$ decays within the same framework of QCD factorization. We find that, contrary to the generalized factorization approach[10], nonfactor-

izable corrections to each helicity amplitude are not the same; the effective parameters a_i^h vary for different helicity amplitudes. The transverse to total decay rate Γ_T/Γ is very small for most decay modes, so in the heavy quark limit, both light vector mesons in charmless $\bar{B}_s \to VV$ decays tend to have zero helicity. Branching ratios for some decay modes are found of order 10^{-5}, which could be measured at LHCb.

This paper is organized as follows. In Sec. II, we outline the necessary ingredients of the QCD factorization approach for describing the $\bar{B}_s \to VV$ decays and calculate the effective parameters a_i^h. Input parameters, numerical calculations and results are presented in Sec. III. Finally we conclude with a summary in Sec. IV. The amplitudes for charmless two-body $\overline{B}_s \to VV$ decays are given in Appendix.

2 $\overline{B}_s \to VV$ in QCD factorization approach

2.1 The effective Hamiltonian

Using the operator product expansion and renormalization group equation, the low energy effective Hamilization relevant to hadronic charmless B decays can be written as [13]

$$\begin{aligned}\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}[\lambda_u(C_1O_1^u + C_2O_2^u) + \lambda_c(C_1O_1^c + C_2O_2^c) \\ -\lambda_t(\sum_{i=3}^{10} C_iO_i + C_{7\gamma}O_{7\gamma} + C_{8g}O_{8g})] + h.c\end{aligned} \tag{1}$$

where $\lambda_i = V_{ib}V_{iq}^*$ are CKM factors and $C_i(\mu)$ are the effective Wilson coefficients which have been reliably evaluated to the next-to-leading logarithmic order. The effective operators O_i can be expressed as follows:

$$\begin{array}{llllll} O_1^u & = & (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, & O_2^u & = & (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \\ O_1^c & = & (\bar{c}b)_{V-A}(\bar{q}c)_{V-A}, & O_2^c & = & (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta c_\alpha)_{V-A}, \\ O_{3(5)} & = & (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A(V+A)}, & O_{4(6)} & = & (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \\ O_{7(9)} & = & \frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V+A(V-A)}, & O_{8(10)} & = & \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}, \\ O_{7\gamma} & = & \frac{e}{8\pi^2}m_b\bar{q}_\alpha\sigma^{\mu\nu}(1+\gamma_5)b_\alpha F_{\mu\nu}, & O_{8g} & = & \frac{g}{8\pi^2}m_b\bar{q}_\alpha\sigma^{\mu\nu}(1+\gamma_5)T^a_{\alpha\beta}b_\beta G^a_{\mu\nu}. \end{array} \tag{2}$$

Where $q = d, s$ and q' denotes all the active quarks at the scale $\mu = \mathcal{O}(m_b)$, i.e., $q' = u, d, s, c, b$.

2.2 The factorizable amplitude for $\overline{B}_s \to VV$

To calculate the decay rate and branching ratios for $\overline{B}_s \to VV$ decays, we need the hadronic matrix element for the local four fermion operators

$$\langle V_1(\lambda_1)V_2(\lambda_2)|(\bar{q}_2 q_3)_{V-A}(\bar{q}_1 b)_{V-A}|\overline{B}_s\rangle, \tag{3}$$

where λ_1, λ_2 are the helicities of the final-state vector mesons V_1 and V_2 with four-momentum p_1 and p_2, respectively. In the rest frame of B_s system, since B_s meson has spin zero, we have $\lambda_1 = \lambda_2 = \lambda$. Let $X^{(B_s V_1, V_2)}$ denote the factorizable amplitude with the vector meson V_2 being factored out, under the naive factorization(NF) approach, we can express $X^{(B_s V_1, V_2)}$ as

$$X^{(B_s V_1, V_2)} = \langle V_2|(\bar{q}_2 q_3)_{V-A}|0\rangle\langle V_1|(\bar{q}_1 b)_{V-A}|\overline{B}_s\rangle. \tag{4}$$

In term of the decay constant and form factors defined by [14, 15, 16]

$$\begin{aligned}
\langle V(p,\varepsilon^*)|\bar{q}\gamma_\mu q'|0\rangle &= -i f_V m_V \varepsilon^*_\mu, &(5)\\
< V(p,\varepsilon^*)|\bar{q}\gamma_\mu(1-\gamma_5)b|\overline{B}_s(p_B) > &= -\varepsilon^*_\mu(m_B+m_V)A_1^{B_s V}(q^2) + (p_B+p)_\mu(\varepsilon^*\cdot p_B)\frac{A_2^{B_s V}(q^2)}{m_B+m_V}\\
&\quad + q_\mu(\varepsilon^*\cdot p_B)\frac{2m_V}{q^2}[A_3^{B_s V}(q^2) - A_0^{B_s V}(q^2)]\\
&\quad - i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha p^\beta\frac{2V^{B_s V}(q^2)}{m_B+m_V}, &(6)
\end{aligned}$$

where $q = p_B - p$ and the form factors obey the following exact relations

$$\begin{aligned}
A_3(0) &= A_0(0),\\
A_3^{B_s V}(q^2) &= \frac{m_B+m_V}{2m_V}A_1^{B_s V}(q^2) - \frac{m_B-m_V}{2m_V}A_2^{B_s V}(q^2).
\end{aligned} \tag{7}$$

With above equations, the factorizable amplitude for $\overline{B}_s \to V_1 V_2$ can be written as

$$\begin{aligned}
X^{(\overline{B}_s V_1, V_2)} &= i f_{V_2} m_{V_2}\Bigg[(\varepsilon_1^*\cdot\varepsilon_2^*)(m_{B_s}+m_{V_1})A_1^{B_s V_1}(m_{V_2}^2) - (\varepsilon_1^*\cdot p_B)(\varepsilon_2^*\cdot p_B)\frac{2A_2^{B_s V_1}(m_{V_2}^2)}{m_{B_s}+m_{V_1}}\\
&\quad + i\epsilon_{\mu\nu\alpha\beta}\varepsilon_2^{*\mu}\varepsilon_1^{*\nu}p_B^\alpha p^\beta\frac{2V^{B_s V}(q^2)}{m_{B_s}+m_{V_1}}\Bigg],
\end{aligned} \tag{8}$$

where $p_B(m_{B_s})$ is the four-momentum(mass) of the $\overline{B}_s$ meson, $m_{V_1}(\varepsilon_1^*)$ and $m_{V_2}(\varepsilon_2^*)$ are the masses(polarization vectors) of the two vector mesons V_1 and V_2 respectively. Here and in the

following throughout the paper we use the sign convention $\epsilon^{0123} = -1$. Assuming the $V_1(V_2)$ meson flying in the plus(minus) z-direction carrying the momentum $p_1(p_2)$, we get

$$X^{(\overline{B}_s V_1, V_2)} = \begin{cases} \frac{-if_{V_2}}{2m_{V_1}}\Big[(m_{B_s}^2 - m_{V_1}^2 - m_{V_2}^2)(m_{B_s}+m_{V_1})A_1^{B_sV_1}(m_{V_2}^2) & \\ -\frac{4m_{B_s}^2 p_c^2}{m_{B_s}+m_{V_1}} A_2^{B_sV_1}(m_{V_2}^2)\Big] \equiv h_0 & \text{for } \lambda = 0, \\ -if_{V_2} m_{V_2}[(m_{B_s}+m_{V_1})A_1^{B_sV_1}(m_{V_2}^2) \pm \frac{2m_{B_s}p_c}{m_{B_s}+m_{V_1}} V^{B_sV_1}(m_{V_2}^2)] \equiv h_\pm & \text{for } \lambda = \pm, \end{cases} \tag{9}$$

where $\lambda = 0, \pm$ is the helicity of the vector meson and $p_c = |\vec{p}_1| = |\vec{p}_2|$ is the momentum of either of the two outgoing vector mesons in the $\overline{B}_s$ rest frame.

In general, the $\overline{B}_s \to V_1 V_2$ amplitude can be decomposed into three independent helicity amplitudes H_0, H_+ and H_-, corresponding to $\lambda = 0$, $+$ and $-$ respectively. We use the notation

$$H_\lambda =< V_1(\lambda)V_2(\lambda)|\mathcal{H}_{eff}|\overline{B}_s > \tag{10}$$

for the helicity matrix element and it can be expressed by three independent *Lorentz* scalars a, b and c. The relations between them can be written as [3, 17]

$$H_\lambda = \varepsilon_{1\mu}^* \varepsilon_{2\nu}^* \left(a g^{\mu\nu} + \frac{b}{m_{V_1} m_{V_2}} p_B^\mu p_B^\nu + \frac{ic}{m_{V_1} m_{V_2}} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right), \tag{11}$$

where the coefficient c corresponds to the p-wave amplitude, a and b to the mixture of s- and d-wave amplitudes. The helicity amplitudes can be reconstructed as

$$H_0 = -ax - b(x^2 - 1), \tag{12}$$

$$H_\pm = -a \mp c\sqrt{x^2 - 1}, \tag{13}$$

where $x = \frac{(p_1 \cdot p_2)}{(m_{V_1} m_{V_2})}$. Given the helicity amplitudes, the decay rate and the branching ratio for $\overline{B}_s \to V_1 V_2$ can be written as

$$\begin{aligned} \Gamma(\overline{B}_s \to V_1 V_2) &= \frac{p_c}{8\pi m_{B_s}^2} \left(|H_0|^2 + |H_+|^2 + |H_-|^2 \right) s, \\ Br(\overline{B}_s \to V_1 V_2) &= \tau_{B_s} \frac{p_c}{8\pi m_{B_s}^2} \left(|H_0|^2 + |H_+|^2 + |H_-|^2 \right) s, \end{aligned} \tag{14}$$

with $s = 1/2$ for two identical final states and $s = 1$ for the other cases. where τ_{B_s} is the lifetime of the B_s meson, and p_c is given by

$$p_c = \frac{1}{2m_{B_s}} \sqrt{[m_{B_s}^2 - (m_{V_1} + m_{V_2})^2][m_{B_s}^2 - (m_{V_1} - m_{V_2})^2]}\ . \tag{15}$$

2.3 QCD factorization for $\bar{B}_s \to VV$

Under the naive factorization(NF) approach, the coefficients a_i are given by $a_i = C_i + \frac{1}{N_C} C_{i+1}$ for odd i and $a_i = C_i + \frac{1}{N_C} C_{i-1}$ for even i, which are obviously independent of the helicity λ. In the present paper, we will compute the nonfactorizable corrections to the effective parameters a_i^h, which however are not the same for different helicity amplitudes H_0 and $H_\pm$.

The QCD-improved factorization(QCDF) approach advocated by Beneke et al. [3] allows us to compute the nonfactorizable corrections to the hadronic matrix elements $\langle V_1 V_2 | O_i | \overline{B}_s \rangle$ in the heavy quark limit, since in the $m_b \to \infty$ limit only hard interactions between the $(\bar{B}_s V_1)$ system and V_2 survive. In this method, the light-cone distribution amplitudes(LCDAs) play an essential role. Since we are only concerned with two light vector mesons in the final states, the LCDAs of the light vector meson of interest in momentum configuration are given by [15, 17]

$$\mathcal{M}^V_{\delta\alpha} = \mathcal{M}^V_{\delta\alpha\parallel} + \mathcal{M}^V_{\delta\alpha\perp} \tag{16}$$

with (here we suppose the vector meson moving in the n_--direction)

$$\begin{aligned}
\mathcal{M}^V_\parallel &= -\frac{i f_V}{4} \frac{m_V (\varepsilon^* \cdot n_+)}{2} \not{n}_- \, \Phi^V_\parallel(u), \\
\mathcal{M}^V_\perp &= -\frac{i f_V^\perp}{4} E \not{\varepsilon}^*_\perp \not{n}_- \, \Phi^V_\perp(u) - \frac{i f_V m_V}{4} \left[\not{\varepsilon}^*_\perp \, g_\perp^{(v)V}(u) + i \epsilon_{\mu\nu\rho\sigma} \varepsilon_\perp^{*\,\nu} n_-^\rho n_+^\sigma \gamma^\mu \gamma_5 \frac{g_\perp^{\prime(a)V}(u)}{8} \right],
\end{aligned} \tag{17}$$

where $n_\pm = (1, 0, 0, \pm 1)$ are the light-cone null vectors, u is the light-cone momentum fraction of the quark in the vector meson, f_V and $f_V^\perp$ are vector and tensor decay constants, and E is the energy of the vector meson in the B_s rest system. In Eq.(17), $\Phi^V_\parallel(u)$ and $\Phi^V_\perp(u)$ are leading-twist distribution amplitudes(DAs), while $g_\perp^{(v)V}(u)$ and $g_\perp^{\prime(a)V}(u) = \frac{\mathrm{d}g_\perp^{(a)V}(u)}{\mathrm{d}u}$ are twist-3 ones. Since the twist-2 DA $\Phi^V_\perp(u)$ contributions to the vertex corrections and hard spectator interactions vanish in the chiral limit, and furthermore, the contributions of the twist-3 DAs $h_\parallel^{(s,t)}(u)$ are power suppressed compared to that of the leading twist ones for the helicity zero case, therefore we will work to the leading-twist approximation for longitudinally polarized states and to the twist-3 level for transversely polarized ones. We note that the same observation has been made by Cheng and Yang [18] in studying $B_{u,d} \to \phi K^*$.

In the heavy quark limit, the light-cone projector for B meson can be expressed as [5, 19]

$$\mathcal{M}^B_{\alpha\beta} = -\frac{i f_B m_B}{4} \left[(1 + \not{v}) \gamma_5 \left\{ \Phi_1^B(\xi) + \not{n}_- \, \Phi_2^B(\xi) \right\} \right]_{\beta\alpha}, \tag{18}$$

with the normalization condition

$$\int_0^1 \mathrm{d}\xi \Phi_1^B(\xi) = 1, \qquad \int_0^1 \mathrm{d}\xi \Phi_2^B(\xi) = 0, \tag{19}$$

where ξ is the momentum fraction of the spectator quark in the B meson.

Equipped with these preliminaries, we can now calculate the nonfactorizable corrections to the effective parameters a_i^h systematically. After direct calculations, we get

$$\begin{aligned}
a_1^h &= C_1 + \frac{C_2}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_2 (f_I^h + f_{II}^h), \\
a_2^h &= C_2 + \frac{C_1}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_1 (f_I^h + f_{II}^h), \\
a_3^h &= C_3 + \frac{C_4}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_4 (f_I^h + f_{II}^h), \\
a_4^h &= C_4 + \frac{C_3}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_3 (f_I^h + f_{II}^h) \\
&\quad + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C}\Bigg\{ (C_3 - \frac{1}{2}C_9)[G^h(s_q) + G^h(s_b) - \begin{pmatrix} \frac{4}{3} \\ \frac{2}{3} \end{pmatrix}] \\
&\quad - C_1 [\frac{\lambda_u}{\lambda_t} G^h(s_u) + \frac{\lambda_c}{\lambda_t} G^h(s_c) + \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}] \\
&\quad + (C_4 + C_6)\sum_{i=u}^{b} G^h(s_i) + \frac{3}{2}(C_8 + C_{10})\sum_{i=u}^{b} G^h(s_i) + C_{8g} G_g^h \Bigg\}, \\
a_5^h &= C_5 + \frac{C_6}{N_C} - \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_6 (f_I^h + f_{II}^h), \\
a_6^h &= C_6 + \frac{C_5}{N_C}, \\
a_7^h &= C_7 + \frac{C_8}{N_C} - \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_8 (f_I^h + f_{II}^h), \\
a_8^h &= C_8 + \frac{C_7}{N_C}, \\
a_9^h &= C_9 + \frac{C_{10}}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_{10} (f_I^h + f_{II}^h), \\
a_{10}^h &= C_{10} + \frac{C_9}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C} C_9 (f_I^h + f_{II}^h),
\end{aligned} \tag{20}$$

where $C_F = (N_C^2 - 1)/2N_C$, and $N_C = 3$ is the number of colors, $s_i = m_i^2/m_b^2$ and $q = d, s$(determined by the $b \to d$ or $b \to s$ transition process). The superscript h denotes the polarization of the vector meson(which is equivalent to λ, but for convenience we shall adopt h in the following) where $h = 0$ denotes helicity 0 state and $h = \pm$ for helicity $\pm$ ones. In the

expression a_4^h, the upper value in parenthesis corresponds to $h = 0$ state, while the lower value to $h = \pm$ ones.

In Eq.(20), f_I^h denotes the contributions from the vertex corrections. In the naive dimensional regularization(NDR) scheme for γ_5, it is given by

$$\begin{aligned}
f_I^0 &= -12\log\frac{\mu}{m_b} - 18 + \int_0^1 \mathrm{d}u\Phi_{\parallel}^{V_2}(u)(3\frac{1-2u}{1-u}\log u - 3i\pi), \\
f_I^{\pm} &= -12\log\frac{\mu}{m_b} - 16 + \int_0^1 \mathrm{d}u[g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4}\zeta]\left\{3\frac{1-2u}{1-u}\log u - 3i\pi)\right. \\
&\quad \left. +2\int_0^1 \mathrm{d}x\mathrm{d}y[\frac{1-x-y}{xy} - \frac{u}{xu+y} \mp \frac{(1-x)u}{y(xu+y)}]\right\}, \qquad (21)
\end{aligned}$$

where $\zeta = +1$ or -1, corresponding to $(V-A)\otimes(V-A)$ or $(V-A)\otimes(V+A)$ current respectively. It is obvious that f_I^0 has the same expression as the hard scattering kernel F_{M_2} for $B \to \pi\pi$ mode [3, 8] as it should be.

For hard spectator interactions, supposing V_1 be the recoiled meson and V_2 the emitted meson, explicit calculations for f_{II}^h yields

$$\begin{aligned}
f_{II}^0 &= -\frac{4\pi^2}{N_C}\frac{if_{B_s}f_{V_1}f_{V_2}}{h_0}\int_0^1 \mathrm{d}\xi\frac{\Phi_1^B(\xi)}{\xi}\int_0^1 \mathrm{d}v\frac{\Phi_{\parallel}^{V_1}(v)}{\bar{v}}\int_0^1 \mathrm{d}u\frac{\Phi_{\parallel}^{V_2}(u)}{u}, \\
f_{II}^{\pm} &= \frac{4\pi^2}{N_C}\frac{if_{B_s}f_{V_1}^{\perp}f_{V_2}m_{V_2}}{m_{B_s}h_{\pm}}2(1\pm1)\int_0^1 \mathrm{d}\xi\frac{\Phi_1^B(\xi)}{\xi}\int_0^1 \mathrm{d}v\frac{\Phi_{\perp}^{V_1}(v)}{\bar{v}^2}\int_0^1 \mathrm{d}u(g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4}\zeta) \\
&\quad -\frac{4\pi^2}{N_C}\frac{if_{B_s}f_{V_1}f_{V_2}m_{V_1}m_{V_2}}{m_{B_s}^2h_{\pm}}\int_0^1 \mathrm{d}\xi\frac{\Phi_1^B(\xi)}{\xi}\int_0^1 \mathrm{d}v\mathrm{d}u(g_{\perp}^{(v)V_1}(v) \mp \frac{g_{\perp}^{\prime(a)V_1}(v)}{4}) \\
&\quad (g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4}\zeta)\frac{u+\bar{v}}{u\bar{v}^2}, \qquad (22)
\end{aligned}$$

with $\bar{v} = 1 - v$, and $h_0, h_{\pm}$ given by Eq. (9). In Eq. (22), when we adopt the asymptotical form for the vector meson LCDAs, there will be a logarithmic infrared divergence with regard to the v integral in $f_{II}^{\pm}$, which implies that the spectator interaction is dominated by soft gluon exchanges in the final states. In analogy with the treatment in works [5, 6, 11], we parameterize it as

$$X_h = \int_0^1 \mathrm{d}x\frac{1}{x} = \log\frac{m_b}{\Lambda_h}(1 + \rho_H e^{i\phi_H}), \qquad (23)$$

with (ρ_H, ϕ_H) related to the contributions from hard spectator scattering. Since the parameters (ρ_H, ϕ_H) are unknown, how to treat them is a major theoretical uncertainty in the QCD factorization approach. In the later numerical analysis, we shall take $\Lambda_h = 0.5GeV$, $(\rho_h, \phi_h) = (0, 0)$ [11] as our default values.

In calculating the contributions of the QCD penguin-type diagrams, we should pay attention to the fact that there are two distinctly different contractions argued in [5]. With this in mind, the nonfactorizable corrections induced by local four-quark operators O_i can be described by the function $G^h(s)$ which is given by

$$\begin{aligned} G^0(s) &= -\frac{2}{3} + \frac{4}{3}\log\frac{\mu}{m_b} - 4\int_0^1 \mathrm{d}u\ \Phi_{\parallel}^{V_2}(u) g(u,s), \\ G^{\pm}(s) &= -\frac{2}{3} + \frac{2}{3}\log\frac{\mu}{m_b} - 2\int_0^1 \mathrm{d}u\ (g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4}) g(u,s), \end{aligned} \tag{24}$$

with the function

$$g(u,s) = \int_0^1 \mathrm{d}x\ x\bar{x}\log[s - x\bar{x}(1-u) - i\epsilon]. \tag{25}$$

In Eq.(20), we also take into account the contributions of the dipole operator O_{8g} which will give a tree-level contribution described by the function G_g^h defined as

$$\begin{aligned} G_g^0 &= \int_0^1 \mathrm{d}u\ \frac{2\Phi_{\parallel}^{V_2}(u)}{1-u}, \\ G_g^+ &= \int_0^1 \mathrm{d}u\ (g_{\perp}^{(v)V_2}(u) - \frac{g_{\perp}^{\prime(a)V_2}(u)}{4}), \\ G_g^- &= \int_0^1 \mathrm{d}u\ (g_{\perp}^{(v)V_2}(u) - \frac{g_{\perp}^{\prime(a)V_2}(u)}{4})\frac{1}{1-u}. \end{aligned} \tag{26}$$

Due to $< V|\bar{q}_1 q_2|0 >= 0$, $\overline{B}_s \to V_1 V_2$ decays do not receive nonfactorizable contributions from a_6^h and a_8^h penguin terms as shown in Eq.(20).

3 Numerical results and discussions

To proceed, we use the next-to-leading order Wilson coefficients in the NDR scheme for γ_5 [11]

$$\begin{aligned} C_1 &= 1.078,\ C_2 = -0.176,\ C_3 = 0.014,\ C_4 = -0.034,\ C_5 = 0.008,\ C_6 = -0.039, \\ C_7/\alpha &= -0.011,\ C_8/\alpha = 0.055,\ C_9/\alpha = -1.341,\ C_{10}/\alpha = 0.264,\ C_{8g} = -0.146. \end{aligned} \tag{27}$$

at $\mu = m_b = 4.66$ GeV, with α being the electromagnetic fine-structure coupling constant. For quark masses, which appears in the penguin loop corrections with regard to the functions $G^h(s)$, we take

$$m_u = m_d = m_s = 0,\ m_c = 1.47\ \text{GeV},\ m_b = 4.66\ \text{GeV}. \tag{28}$$

As for the CKM matrix elements, we adopt the Wolfenstein parametrization up to $\mathcal{O}(\lambda^3)$:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{29}$$

for the Wolfenstein parameters appearing in the above expression, we shall use the values given by [20]

$$\lambda = 0.2236,\ A = 0.824,\ \bar{\rho} = 0.22,\ \bar{\eta} = 0.35, \tag{30}$$

where $\bar{\rho} = \rho(1-\frac{\lambda^2}{2})$ and $\bar{\eta} = \eta(1-\frac{\lambda^2}{2})$. For computing the branching ratio, the lifetime of B_s meson is $\tau_{B_s} = 1.461$ ps [20].

For the LCDAs of the vector meson, we use the asymptotic form [17]

$$\begin{aligned} \Phi_{\parallel}^V(x) &= \Phi_{\perp}^V(x) = g_{\perp}^{(a)V} = 6x(1-x), \\ g_{\perp}^{(v)V}(x) &= \frac{3}{4}[1+(2x-1)^2]. \end{aligned} \tag{31}$$

As for the two B_s meson wave functions given by Eq.(18), we find that only $\Phi_1^B(\xi)$ has contributions to the nonfactorizable corrections. We adopt the moments of the $\Phi_1^B(\xi)$ defined by [3, 5] in our numerical evaluation

$$\int_0^1 \mathrm{d}\xi \frac{\Phi_1^B(\xi)}{\xi} = \frac{m_{B_s}}{\Lambda_B}, \tag{32}$$

with $\Lambda_B = 0.35$ GeV. The quantity Λ_B parameterizes our ignorance about the B_s meson distribution amplitudes and thus brings large theoretical uncertainty.

The decay constants and form factors are nonperturbative parameters which are taken as input parameters. In principle, they are available from the experimental data and /or estimated with well-founded theories, such as lattice calculations, QCD sum rules etc. For the decay constants, we take their values in our calculations as [5, 11, 21, 14]

$$\begin{aligned} &f_{B_s} = 236\text{ MeV},\ f_{K^*} = 214\text{ MeV},\ f_{K^*}^{\perp} = 175\text{ MeV},\ f_{\rho} = 210\text{ MeV}, \\ &f_{\omega} = 195\text{ MeV},\ f_{\phi} = 233\text{ MeV},\ f_{\phi}^{\perp} = 175\text{ MeV}. \end{aligned} \tag{33}$$

For the form factors involving the $B_s \to K^*$ and $B_s \to \phi$ transition, we adopt the results given by [14] which are analyzed using the light-cone sum rule(LCSR) method with the parameterization

$$f(q^2) = \frac{f(0)}{1 - a_F(q^2/m_{B_s}^2) + b_F(q^2/m_{B_s}^2)^2} \tag{34}$$

for the form-factor q^2 dependence. At the maximum recoil, the form factors are listed as [14]

$$\begin{aligned} A_1^{B_s\phi}(0) &= 0.296, \quad a_F = 0.87, \quad b_F = -0.061, \\ A_2^{B_s\phi}(0) &= 0.255, \quad a_F = 1.55, \quad b_F = 0.513, \\ V^{B_s\phi}(0) &= 0.433, \quad a_F = 1.75, \quad b_F = 0.736, \\ A_1^{B_sK^*}(0) &= 0.190, \quad a_F = 1.02, \quad b_F = -0.037, \\ A_2^{B_sK^*}(0) &= 0.164, \quad a_F = 1.77, \quad b_F = 0.729, \\ V^{B_sK^*}(0) &= 0.262, \quad a_F = 1.89, \quad b_F = 0.846. \end{aligned} \tag{35}$$

It is obvious that the q^2 dependence for the form factors A_2 and V are dominated by the dipole terms, while A_1 by the monopole term in the region where q^2 is not too large.

To illustrate the non-universality of the nonfactorizable effects on different helicity amplitudes, we list a few numerical results of the parameters a_i^h for a specific mode $\overline{B}_s \to K^{+*}\rho^-$ in Table 1. In order to compare with the parameters a_i in the NF approach, we also present the results of a_i calculated in NF approach.

Table 1: The effective parameters a_i^h in the NF and QCDF approach for $\bar{B}_s \to K^{+*}\rho^-$.

a_i^h	NF	QCDF
a_1^0	1.0193	$1.0265 + 0.0126i$
a_4^0	-0.0293	$-0.0263 - 0.0015i$
a_{10}^0	-0.0013	$-0.0009 + 0.0007i$
a_1^+	1.0193	$1.0701 + 0.0126i$
a_4^+	-0.0293	$-0.0385 - 0.0015i$
a_{10}^+	-0.0013	$0.0015 + 0.0007i$
a_1^-	1.0193	$1.0943 + 0.0126i$
a_4^-	-0.0293	$-0.0374 + 0.0022i$
a_{10}^-	-0.0013	$0.0028 + 0.0007i$

From Table 1, we can see that nonfactorizable corrections to the helicity amplitudes are not universal. The effective parameters a_i^h for helicity $h = 0, +, -$ states receive different

nonfactorizable contributions and hence they are helicity dependent, quite contrary to the naive factorization(NF) approach where the parameters a_i are universal and polarization independent.

The branching ratios for several channels of $\overline{B}_s \to VV$ decays in the LCSR analysis for form factors are collected in Table 2. In order to compare the size of different helicity amplitudes, we define two quantities:

$$\frac{\Gamma_T}{\Gamma} = \frac{|H_+|^2+|H_-|^2}{|H_0|^2+|H_+|^2+|H_-|^2}, \tag{36}$$

$$\frac{\Gamma_L}{\Gamma} = \frac{|H_+|^2+|H_-|^2}{|H_0|^2+|H_+|^2+|H_-|^2}. \tag{37}$$

The ratios of Γ_T/Γ and Γ_L/Γ measure the relative amount of transversely and longitudinally polarized vector meson. In Table II, we also give the values of Γ_T/Γ for each channel both in QCD factorization(QCDF) approach and the naive factorization(NF) approach.

Table 2: Branching ratios and the transverse to total decay rate Γ_T/Γ for charmless $\bar{B}_s \to VV$ decays in QCD factorization(QCDF) approach and in the NF approach.

channel	Γ_T/Γ		BR	
	QCDF	NF	QCDF	NF
$b \to d$ transition				
$\overline{B}_s \to K^{+*}\rho^-$	0.071	0.066	1.82×10^{-5}	1.79×10^{-5}
$\overline{B}_s \to K^{0*}\rho^0$	0.072	0.062	5.29×10^{-7}	5.94×10^{-7}
$\overline{B}_s \to K^{0*}\omega$	0.046	0.064	3.08×10^{-7}	7.32×10^{-7}
$b \to s$ transition				
$\overline{B}_s \to K^{+*}K^{-*}$	0.100	0.103	1.94×10^{-6}	1.58×10^{-6}
$\overline{B}_s \to \omega\phi$	0.141	0.072	5.31×10^{-7}	2.64×10^{-7}
$\overline{B}_s \to \rho^0\phi$	0.089	0.070	1.03×10^{-6}	7.14×10^{-7}
pure penguin processes				
$\overline{B}_s \to K^{0*}\overline{K}^{0*}$	0.094	0.082	2.61×10^{-6}	1.94×10^{-6}
$\overline{B}_s \to K^{0*}\phi$	0.190	0.092	1.35×10^{-7}	1.41×10^{-7}
$\overline{B}_s \to \phi\phi$	0.134	0.117	1.31×10^{-5}	9.05×10^{-6}

From Table 2, we can find that some channels have large branching ratios of order 10^{-5},

which are measurable at near future experiments at CERN LHCb. Owing to the absence of $(S-P)(S+P)$ penguin operator contributions to W-emission amplitudes, tree-dominated $\bar{B}_s \to V_1V_2$ decays tend to have larger branching ratios than the penguin-dominated ones. Moreover, we find that the transverse to total decay rate Γ_T/Γ is very small for most decay modes, so in the heavy quark limit, both light vector mesons in charmless $\overline{B}_s \to VV$ decays tend to have zero helicity.

4 Summary

In this paper, we calculated the branching ratios for two-body charmless hadronic $\overline{B}_s \to VV$ decays within the framework of QCD factorization. Contrary to phenomenological generalized factorization[9] and NF approach, the nonfactorizable corrections to each helicity amplitude are not the same. The effective parameters a_i^h vary for different helicity amplitude and hence are helicity dependent. Since the leading-twist DAs contributions to the transversely polarized amplitudes vanish in the chiral limit, in order to have renormalization scale and scheme independent predictions, it is necessary to take into account the contributions of the twist-3 DAs of the vector meson. Contrary to the PP and PV modes, the annihilation amplitudes in the VV case do not gain the chiral enhancement of order $m_B^2/(m_q m_b)$. So we do not include the contributions of the annihilation diagrams which is truly power suppressed in the heavy quark limit. It should be stressed that we have not taken into account the higher-twist DAs contribution for the longitudinally polarized vector meson. Though direct calculation, the transverse to total decay rate Γ_T/Γ is found to be very small, and both light vector mesons tend to have zero helicity. Branching ratios of $\bar{B}_s \to VV$ decays are calculated with the LCSR analysis for the form factors and the branching ratios of some channels are found as large as 10^{-5}, which might be accessible at future experiments at CERN LHCb.

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Appendix.

The $\overline{B_s} \to VV$ decay amplitudes are collected here:

1. $b \to d$ processes:

$$\begin{aligned}
H^h(\overline{B}_s \to K^{+*}\rho^-) &= \frac{G_F}{\sqrt{2}}\left\{\lambda_u a_1^h - \lambda_t(a_4^h + a_{10}^h)\right\} X^{(\overline{B}_s K^{+*},\rho^-)} \\
H^h(\overline{B}_s \to K^{0*}\rho^0) &= \frac{G_F}{\sqrt{2}}\left\{\lambda_u a_2^h - \lambda_t(-a_4^h + \frac{3}{2}a_7^h + \frac{3}{2}a_9^h + \frac{1}{2}a_{10}^h)\right\} X^{(\overline{B}_s K^{0*},\rho^0)} \\
H^h(\overline{B}_s \to K^{0*}\omega) &= \frac{G_F}{\sqrt{2}}\left\{\lambda_u a_2^h - \lambda_t(2a_3^h + a_4^h + 2a_5^h \right. \\
&\left. + \frac{1}{2}a_7^h + \frac{1}{2}a_9^h - \frac{1}{2}a_{10}^h)\right\} X^{(\overline{B}_s K^{0*},\omega)}. \qquad (38)
\end{aligned}$$

where $\lambda_u = V_{ub}V_{ud}^*$ and $\lambda_t = V_{tb}V_{td}^*$

2. $b \to s$ processes:

$$\begin{aligned}
H^h(\overline{B}_s \to K^{+*}K^{-*}) &= \frac{G_F}{\sqrt{2}}\left\{\lambda_u a_1^h - \lambda_t(a_4^h + a_{10}^h)\right\} X^{(\overline{B}_s K^{+*},K^{-*})} \\
H^h(\overline{B}_s \to \rho^0\phi) &= \frac{G_F}{\sqrt{2}}\left\{\lambda_u a_2^h - \lambda_t[\frac{3}{2}(a_7^h + a_9^h)]\right\} X^{(\overline{B}_s \phi,\rho^0)} \\
H^h(\overline{B}_s \to \omega\phi) &= \frac{G_F}{\sqrt{2}}\left\{\lambda_u a_2^h - \lambda_t[2(a_3^h + a_5^h) + \frac{1}{2}(a_7^h + a_9^h)]\right\} X^{(\overline{B}_s \phi,\omega)}. \qquad (39)
\end{aligned}$$

where $\lambda_u = V_{ub}V_{us}^*$ and $\lambda_t = V_{tb}V_{ts}^*$

3. pure penguin processes:

$$\begin{aligned}
H^h(\overline{B}_s \to K^{0*}\overline{K}^{0*}) &= -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*(a_4^h - \frac{1}{2}a_{10}^h)X^{(\overline{B}_s K^{0*},\overline{K}^{0*})} \\
H^h(\overline{B}_s \to K^{0*}\phi) &= -\frac{G_F}{\sqrt{2}}V_{tb}V_{td}^*\left\{[a_3^h + a_5^h - \frac{1}{2}(a_7^h + a_9^h)]X^{(\overline{B}_s K^{0*},\phi)} \right. \\
&\left. +(a_4^h - \frac{1}{2}a_{10}^h)X^{(B_s\phi,K^{0*})}\right\} \\
H^h(\overline{B}_s \to \phi\phi) &= -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*[a_3^h + a_4^h + a_5^h - \frac{1}{2}(a_7^h + a_9^h + a_{10}^h)]X^{(\overline{B}_s\phi,\phi)}. \qquad (40)
\end{aligned}$$

In the above expressions, the factorozable amplitude $X^{(\overline{B}_s V_1,V_2)}$ is defined as in Eq(8).